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Neutrino spin evolution in presence of general external fields

Maxim Dvornikov and Alexander Studenikin

Department of Theoretical Physics, Moscow State University
119992 Moscow, Russia
E-mail: maxim_dvornikov@aport.ru, studenik@srd.sinp.msu.ru

Abstract: The derivation of the quasiclassical Lorentz invariant neutrino spin evolution equation taking into account general types of neutrino non-derivative interactions with external fields is presented. We discuss the constraints on the characteristics of matter and neutrino under which this quasiclassical approach is valid. The application of the obtained equation for the case of the standard model neutrino interactions with moving and polarized background matter is considered.

Keywords: standard model, Neutrino Physics.

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It is commonly believed that neutrino physics provides strong evidence for physics beyond the standard model. In different extensions of the standard model new types of interactions are predicted for massive neutrinos. The problem of neutrino propagation in matter in the case of a general set of interactions with the background fermions has attracted considerable attention (see, for example [1, 2]). Recently, we have developed the Lorentz invariant formalism for description of neutrino spin-flavor and flavor oscillations with the standard model vector and axial-vector interactions in moving matter under the influence of an arbitrary electromagnetic fields [3, 4, 5]. In particular, we have derived, within general assumption like Lorentz invariance and linearity over neutrino spin vector S^{μ} and also over such characteristics of matter like fermions currents and polarizations, the evolution equation for the neutrino spin. We have used this evolution equation for description of neutrino oscillations in electromagnetic fields accounting for neutrino vector and axial-vector interactions with background fermions that corresponds to the case of the standard model weak interactions. Note that quasiclassical approach to the problem of neutrino spin relaxation in stochastic electromagnetic fields was used in ref. [6]. However, the problem of the neutrino spin evolution equation accounting also for more general new types of neutrino interactions is still remained open.

We discuss below neutrino spin evolution in background matter in the case of a new physics model that admits a general set of new neutrino interactions. The goal of this paper is to derive the neutrino spin evolution equation starting directly from the neutrino interaction lagrangian. We suppose that the neutrino interaction lagrangian includes not only the standard model vector and axial-vector terms, but also non-derivative scalar, pseudoscalar, tensor and pseudotensor interactions.

The derivation of the neutrino spin evolution equation presented here is based on general spin evolution equation in the Heisenberg representation. This approach allows us to analyse more attentively contributions to the neutrino spin evolution of different mentioned above external fields.

To derive the neutrino quasiclassical spin evolution equation we start with the quantum equation in the Heisenberg representation which describes spin evolution of a fermion having an energy E_ν, momentum $\vec{p}$ and mass m_ν (see, for instance, [7])

$$\dot{\vec{O}} = i\left[\mathcal{H}, \vec{O}\right]_{-}. \tag{1}$$

The spin operator is determined as

$$\vec{O} = \rho_3 \vec{\Sigma} + \rho_1 \frac{\vec{p}}{E_\nu} - \rho_3 \frac{\vec{p}\left(\vec{p}\vec{\Sigma}\right)}{E_\nu(E_\nu + m_\nu)}, \tag{2}$$

where $\rho_1 = -\gamma^5$, $\rho_3 = \gamma^0$, $\vec{\Sigma} = \gamma^0\gamma^5\vec{\gamma}$. Note that here we take $\hbar = c = 1$. The hamiltonian $\mathcal{H}$ in eq. (1) describes time behavior of four-component neutrino wave function $\nu(x)$.

The lagrangian $\mathcal{L}$ accounting for general types of neutrino non-derivative interactions with external fields is chosen in following form,

$$
\begin{aligned}
-\mathcal{L} = {} & g_s s(x)\bar{\nu}\nu + g_p \pi(x)\bar{\nu}\gamma^5\nu + g_v V^\mu(x)\bar{\nu}\gamma_\mu\nu + g_a A^\mu(x)\bar{\nu}\gamma_\mu\gamma^5\nu + \\
& + \frac{g_t}{2} T^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\nu + \frac{g_t^{\prime}}{2}\Pi^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\gamma_5\nu\,,
\end{aligned}
\tag{3}
$$

where $s, \pi, V^\mu = (V^0, \vec{V}), A^\mu = (A^0, \vec{A}), T_{\mu\nu} = (\vec{a}, \vec{b}), \Pi_{\mu\nu} = (\vec{c}, \vec{d})$ are the scalar, pseudoscalar, vector, axial-vector, tensor, pseudotensor fields, respectively, and g_i ($i = a, p, v, a, t, t'$) are corresponding coupling constants, $\sigma_{\mu\nu} = i/2(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$. With the use of the lagrangian of eq. (3) the expression for the hamiltonian $\mathcal{H}$ is straightforward:

$$
\begin{aligned}
\mathcal{H} = {} & g_s s\rho_3 - i g_p \pi\rho_2 + g_v\left(V^0 - \left(\vec{\alpha}\vec{V}\right)\right) - g_a\left(\rho_1 A^0 - \left(\vec{\Sigma}\vec{A}\right)\right) - \\
& - g_t\left(\rho_3\left(\vec{\Sigma}\vec{b}\right) + \rho_2\left(\vec{\Sigma}\vec{a}\right)\right) - i g_t^{\prime}\left(\rho_3\left(\vec{\Sigma}\vec{c}\right) - \rho_2\left(\vec{\Sigma}\vec{d}\right)\right),
\end{aligned}
\tag{4}
$$

where $\vec{\alpha} = \gamma^0\vec{\gamma}$, $\rho_2 = i\rho_1\rho_3$. Then, from eq. (1) we obtain:

$$
\begin{aligned}
\dot{\vec{O}} = {} & -2g_s s\frac{\vec{p}}{E_\nu} + 2ig_p\pi\left\{\rho_1\vec{\Sigma} - \rho_3\frac{\vec{p}}{E_\nu} - \rho_1\frac{\vec{p}\left(\vec{p}\vec{\Sigma}\right)}{E_\nu(E_\nu + m_\nu)}\right\} - \\
& - 2g_v\rho_2\left\{\vec{V} - \frac{\vec{p}\left(\vec{p}\vec{V}\right)}{E_\nu(E_\nu + m_\nu)}\right\} - \\
& - 2g_a\left\{A^0\rho_2\left(\vec{\Sigma} - \frac{\vec{p}\left(\vec{p}\vec{\Sigma}\right)}{E_\nu(E_\nu + m_\nu)}\right) - \rho_3\left(\vec{A}\times\vec{\Sigma} + \frac{\vec{p}\left(\vec{A}\left[\vec{p}\times\vec{\Sigma}\right]\right)}{E_\nu(E_\nu + m_\nu)}\right)\right\} + \\
& + 2g_t\left\{\left[\vec{\Sigma}\times\vec{b}\right] + \rho_2\frac{\vec{p}}{E_\nu}\left(\vec{\Sigma}\vec{b}\right) + \frac{\vec{p}\left(\vec{\Sigma}\left[\vec{p}\times\vec{b}\right]\right)}{E_\nu(E_\nu + m_\nu)} + \right. \\
& \qquad \left. + \rho_1\left(\vec{a} - \frac{\vec{p}(\vec{p}\vec{a})}{E_\nu(E_\nu + m_\nu)}\right) - \rho_3\frac{\vec{p}}{E_\nu}\left(\vec{\Sigma}\vec{a}\right)\right\} + \\
& + 2ig_t^{\prime}\left\{\left[\vec{\Sigma}\times\vec{c}\right] + \rho_2\frac{\vec{p}}{E_\nu}\left(\vec{\Sigma}\vec{c}\right) + \frac{\vec{p}\left(\vec{\Sigma}\left[\vec{p}\times\vec{c}\right]\right)}{E_\nu(E_\nu + m_\nu)} - \right. \\
& \qquad \left. - \rho_1\left(\vec{d} - \frac{\vec{p}\left(\vec{p}\vec{d}\right)}{E_\nu(E_\nu + m_\nu)}\right) + \rho_3\frac{\vec{p}}{E_\nu}\left(\vec{\Sigma}\vec{d}\right)\right\}.
\end{aligned}
\tag{5}
$$

In getting this equation we suppose that all external fields are independent of spatial coordinates.

It should be noted that the equation obtained does not seem to have classical interpretation because of the *zitterbewegung* [8]. To eliminate this phenomenon we extract, following the recipe of the ref. [7], an even part from eq. (5):

$$
\left\{\dot{\vec{O}}\right\} = \frac{1}{2E_\nu}\left\{\dot{\vec{O}}, \mathcal{H}_0\right\}_+,
\tag{6}
$$

where $\mathcal{H}_0 = \vec{\alpha}\vec{p} + \rho_3 m_\nu$.

Performing anticommutations in eq. (6) we get the following expression:

$$\left\{\dot{\vec{O}}\right\} = 2g_a\left(\frac{A^0}{E_\nu}\left[\vec{O}\times\vec{p}\right] - \frac{m_\nu}{E_\nu}\left[\vec{O}\times\vec{A}\right] - \frac{1}{E_\nu(E_\nu+m_\nu)}\left(\vec{A}\vec{p}\right)\left[\vec{O}\times\vec{p}\right]\right) +$$
$$+ 2g_t\left(\left[\vec{O}\times\vec{b}\right] - \frac{1}{E_\nu(E_\nu+m_\nu)}\left(\vec{p}\vec{b}\right)\left[\vec{O}\times\vec{p}\right] + \frac{1}{E_\nu}\left[\vec{O}\times[\vec{a}\times\vec{p}]\right]\right) +$$
$$+ 2ig_t'\left(\left[\vec{O}\times\vec{c}\right] - \frac{1}{E_\nu(E_\nu+m_\nu)}(\vec{p}\vec{c})\left[\vec{O}\times\vec{p}\right] - \frac{1}{E_\nu}\left[\vec{O}\times\left[\vec{d}\times\vec{p}\right]\right]\right). \qquad (7)$$

In the quasiclassical approximation ($\hbar \to 0$) for any operator $\hat{F}$ one has $\dot{\hat{F}} = \{\dot{\hat{F}}\}$ [7]. That is why in order to obtain the quasiclassical equation for the neutrino spin evolution we substitute $\{\dot{\vec{O}}\}$ for $\dot{\vec{O}}$. Then, averaging the relevant equation over the stationary neutrino wave function and taking into account that

$$\left\langle\vec{O}\right\rangle = \vec{\zeta}_\nu\,, \qquad \langle\vec{p}\rangle = \vec{\beta}E_\nu\,,$$

where $\vec{\beta}$ is the neutrino velocity, we obtain the relativistic equation for the neutrino spin vector $\vec{\zeta}_\nu$ in the form

$$\dot{\vec{\zeta}}_\nu = 2g_a\left\{A^0\left[\vec{\zeta}_\nu\times\vec{\beta}\right] - \frac{m_\nu}{E_\nu}\left[\vec{\zeta}_\nu\times\vec{A}\right] - \frac{E_\nu}{E_\nu+m_\nu}\left(\vec{A}\vec{\beta}\right)\left[\vec{\zeta}_\nu\times\vec{\beta}\right]\right\} +$$
$$+ 2g_t\left\{\left[\vec{\zeta}_\nu\times\vec{b}\right] - \frac{E_\nu}{E_\nu+m_\nu}\left(\vec{\beta}\vec{b}\right)\left[\vec{\zeta}_\nu\times\vec{\beta}\right] + \left[\vec{\zeta}_\nu\times\left[\vec{a}\times\vec{\beta}\right]\right]\right\} +$$
$$+ 2ig_t'\left\{\left[\vec{\zeta}_\nu\times\vec{c}\right] - \frac{E_\nu}{E_\nu+m_\nu}\left(\vec{\beta}\vec{c}\right)\left[\vec{\zeta}_\nu\times\vec{\beta}\right] - \left[\vec{\zeta}_\nu\times\left[\vec{d}\times\vec{\beta}\right]\right]\right\}. \qquad (8)$$

It is worth to be noted that in agreement with [2] neither scalar nor pseudoscalar nor vector interaction contributes to the neutrino spin evolution.

The Lorenz invariant form of eq. (8) can be obtained using the four-dimensional spin vector S^μ which is determined by the three-dimensional spin vector $\vec{\zeta}_\nu$ in accordance with the relation:

$$S^\mu = \left(\frac{\left(\vec{\zeta}_\nu\langle\vec{p}\rangle\right)}{m_\nu}, \vec{\zeta}_\nu + \frac{\langle\vec{p}\rangle\left(\vec{\zeta}_\nu\langle\vec{p}\rangle\right)}{m_\nu(m_\nu+E_\nu)}\right). \qquad (9)$$

Thus, we get the Lorentz invariant form for the neutrino spin S^μ evolution equation accounting for the general interactions with external fields

$$\frac{dS^\mu}{d\tau} = 2g_t\left(T^{\mu\nu}S_\nu - u^\mu T^{\lambda\rho}u_\lambda S_\rho\right) + 2ig_t'\left(\tilde{\Pi}^{\mu\nu}S_\nu - u^\mu\tilde{\Pi}^{\lambda\rho}u_\lambda S_\rho\right) + 2g_a G^{\mu\nu}S_\nu\,, \qquad (10)$$

where $G^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}A_\alpha u_\beta$, $u^\mu = E_\nu/m_\nu(1,\vec{\beta})$, $\tilde{\Pi}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\Pi_{\alpha\beta}$. The tensor $G_{\mu\nu}$ can be expressed through two vectors $G_{\mu\nu} = (-\vec{P},\vec{M})$ which are presented in the form,

$$\vec{M} = \gamma\left(A^0\vec{\beta} - \vec{A}\right),$$
$$\vec{P} = -\gamma\left[\vec{\beta}\times\vec{A}\right], \qquad (11)$$

where $\gamma = E_\nu/m_\nu$. The derivation in the left-handed side of eq. (10) is taken over the neutrino proper time $\tau = \frac{m_\nu}{E_\nu}t$, where t is the time in the laboratory frame of reference.

The neutrino spin evolution eq. (10) can be used for any theoretical model in which neutrino has mentioned above general interactions. Let us consider now the case of the standard model neutrino interactions which are sure to be one of the possible applications of the approach concerned and suppose that matter is composed of electrons, neutrons and protons. Then, the coupling constants entering in the lagrangian of eq. (3) are $g_i = 0$ (for $i = s, p, t'$), $g_v = g_a = G_F/\sqrt{2}$, $g_t = \mu$, where G_F is the Fermi constant and μ is the neutrino magnetic moment. The vector and axial-vector fields are expressed in the form (see, for instance [9])

$$A^\mu = V^\mu = \sum_{f=e,p,n} j_f^\mu q_f^{(1)} + \lambda_f^\mu q_f^{(2)}\,, \tag{12}$$

where

$$\begin{aligned} q_f^{(1)} &= \left(I_{3L}^{(f)} - 2Q^{(f)} \sin^2\theta_W + \delta_{ef}\right), \qquad q_f^{(2)} = -\left(I_{3L}^{(f)} + \delta_{ef}\right), \\ \delta_{ef} &= \begin{cases} 1\,, & f = e\,, \\ 0\,, & f = n, p \end{cases} \end{aligned} \tag{13}$$

and $I_{3L}^{(f)}$ is the value of the isospin third component of a fermion f, $Q^{(f)}$ is the value of its electric charge and θ_W is the Weinberg angle. In the case of the standard model the tensor field corresponds to the electromagnetic interaction and

$$T_{\mu\nu} = F_{\mu\nu} = \left(\vec{E}, \vec{B}\right), \tag{14}$$

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields, respectively. The summation in eq. (12) is performed over the background electrons, protons and neutrons. Equation (12) for external field A^μ depends upon fermions currents j_f^μ and polarizations λ_f^μ which are related with the matter components number densities n_f, speeds $\vec{v}_f$ of the reference frames in which the mean momentum of each of the fermions $f = e, p, n$ equals zero and the mean value of the fermions polarization vectors $\vec{\zeta}_f$:

$$\begin{aligned} j_f^\mu &= (n_f, n_f\vec{v}_f)\,, \\ \lambda_f^\mu &= \left(n_f\left(\vec{\zeta}_f\vec{v}_f\right), n_f\vec{\zeta}_f\sqrt{1-v_f^2} + \frac{n_f\vec{v}_f\left(\vec{\zeta}_f\vec{v}_f\right)}{1+\sqrt{1-v_f^2}}\right). \end{aligned} \tag{15}$$

The details of the averaging procedure over the background fermions are discussed in ref. [4].

The covariant neutrino spin evolution equation with the external fields defined by eqs. (10)–(15) enables one to describe the neutrino spin precession in the case of the standard model in the arbitrary moving and polarized matter, with the neutrino mass accounted exactly.[1] On the base of the eq. (10) (see also (8)) let us now consider the equation for the neutrino spin vector $\vec{\zeta}_\nu$,

$$\dot{\vec{\zeta}}_\nu = \frac{2\mu}{\gamma}\left[\vec{\zeta}_\nu \times \vec{B}_0\right] + \frac{\sqrt{2}G_F}{\gamma}\left[\vec{\zeta}_\nu \times \vec{M}_0\right], \tag{16}$$

[1]Our approach also does not imply the use of the assumption $E_\nu \gg m_\nu$.

where

$$\vec{M}_0 = \gamma\vec{\beta}\left(A^0 - \frac{E_\nu}{E_\nu + m_\nu}\left(\vec{A}\vec{\beta}\right)\right) - \vec{A},$$
$$\vec{B}_0 = \gamma\left\{\vec{B} + \left[\vec{E}\times\vec{\beta}\right] - \frac{E_\nu}{E_\nu + m_\nu}\vec{\beta}\left(\vec{\beta}\vec{B}\right)\right\}, \tag{17}$$

are the quantities determined in the neutrino rest frame. The derived eqs. (16) and (17) reproduce the spin evolution equations for neutrinos propagating in the background electrons (the electron plasma) which were received in ref. [4] on the base of the generalization of the Bargmann-Michel-Telegdi equation. We underline that the approach considered here implies the use as a starting point the *Lorentz invariant* neutrino interaction lagrangian for derivation of the *Lorentz invariant* spin evolution equation.

It is worth mentioning that neutrino spin evolution problem in presence of the background fermions can be considered using kinetic equations approach [10]. The analog of the eqs. (16) and (17) for the specific cases of non moving ($\vec{v}_f = 0$) and unpolarized ($\vec{\zeta}_f = 0$) medium was shown to be derived from the more general kinetic equation for the very important case of the ultrarelativistic neutrino. However our approach, as it was mentioned above, is valid in cases of arbitrary speed of neutrino and arbitrary moving and polarized background matter.

Let us discuss in some details approximations which we use in deriving the neutrino spin evolution equation. First we have neglected spatial coordinate dependence of all external fields. To analyze the adequacy of this approximation we now consider the opposite case. For simplicity, we shall discuss the standard model neutrino interactions in the case of non moving and unpolarized background matter. Then, the hamiltonian (4) takes the form,

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} n_{\text{eff}}(1+\gamma_5), \tag{18}$$

where $n_{\text{eff}} = \sum_{f=e,p,n} n_f q_f^{(1)}$ is an effective density of the background matter which is now supposed to depend on spatial coordinates. To study corrections to eq. (8) induced by the spatial dependence of n_{eff} we must take into account the commutation relation, $[n_{\text{eff}}, \vec{p}]_- = i\hbar\vec{\nabla}n_{\text{eff}}$. Contrary to the used above agrement here we set $\hbar \neq 1$. Thus, holding the first order corrections in expansion over $\hbar$ we obtain an analog of eq. (8),

$$\dot{\vec{\zeta}}_\nu = \frac{\sqrt{2}G_F}{\hbar}\left\{ n_{\text{eff}}\left[\vec{\zeta}_\nu\times\vec{\beta}\right] - \frac{\hbar}{2(E_\nu+m_\nu)}\left[\left[\left[\vec{\beta}\times\vec{\nabla}n_{\text{eff}}\right]\times\vec{\zeta}_\nu\right] + \right.\right.$$
$$\left.\left. + \left(\vec{\nabla}n_{\text{eff}}\vec{\beta}\right)\left(\frac{E_\nu}{E_\nu+m_\nu}\left(\vec{\beta}\vec{\zeta}_\nu\right) - 1\right)\right] - \frac{i\hbar}{2E}\left[\vec{\nabla}n_{\text{eff}}\times\left\langle\rho_3\vec{\Sigma}\right\rangle\right]\right\}, \tag{19}$$

where the average in the last form $\langle\rho_3\vec{\Sigma}\rangle = \int d^3\mathbf{x}\nu^+\rho_3\vec{\Sigma}\nu$. It follows that for the relativistic neutrinos ($E_\nu \gg m_\nu$) the additional quantum terms ($\sim\hbar$) vanish if the following constrained is fulfilled

$$\frac{\hbar}{E_\nu}\left|\frac{\vec{\nabla}n_{\text{eff}}}{n_{\text{eff}}}\right| \ll 1. \tag{20}$$

This condition implies very slow effective density variation along the neutrino wave package width $L = \hbar/E_\nu$.

Let us discuss the second approximation made in the derivation of eq. (8) which enables us to perform calculations in the quasiclassical limit. In order to neglect the *zitterbewegung* we must satisfy the condition [7]:

$$\frac{\hbar}{2E_\nu}\left|\left\langle \dot{\vec{O}}\right\rangle\right| \ll 1\,. \tag{21}$$

In application to the case of the standard model interactions it can be rewritten as

$$\frac{G_F}{E_\nu}n_{\rm eff} \ll 1\,. \tag{22}$$

Here again the background matter is taken to be non moving and unpolarized. This condition means that quantum effects are not important in the neutrino scattering off the background matter. Indeed, if, for simplicity, we consider only electron component of the background matter (the electron plasma) the constraint (22) is expressed in the following way:

$$L^2 \ll \lambda\sqrt{\sigma}\,, \tag{23}$$

where σ is the neutrino-electron cross section, $\lambda \sim 1/n_e\sigma$ is the mean free path of neutrino in this medium, n_e is the electrons density. If the condition (23) is satisfied, the characteristic dimensions, λ and σ, corresponding to the scattering process are much larger than neutrino wave package width L. In the opposite case the neutrino-electron scattering have to be treated within the substantially quantum approach.

In summary we have derived the quasiclassical Lorentz invariant equation for the neutrino spin evolution in the case when neutrinos interact with external fields through a set of the most general non-derivative interactions starting from the neutrino lagrangian that accounts for the scalar, pseudoscalar, vector, axial-vector, tensor, pseudotensor terms. The advantage of the suggested new method for derivation of the neutrino spin evolution equation is manifested by the applicability to any theoretical model which predicts the mentioned above neutrino interactions. Furthermore the approach used enables us to examine the constraints for the characteristics of matter and neutrino under which the quasiclassical approach to the neutrino spin evolution is valid. As an example we have obtained the neutrino spin evolution equation for the standard model neutrino interactions in the case of moving and polarized matter.

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